Worksheet for September 26

Problems marked with an asterisk are to be placed in your math diary.

(1.*) For the function $f(x,y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0), \end{cases}$ show that:

(i) $f(x,y)$ is continuous at (0,0).
(ii) $f_x(x,y)$ and $f_y(x,y)$ are continuous at (0,0).
(iii) $f_{xy}(0,0)$ and $f_{yx}(0,0)$ exist, but are not equal.
(iv) Show that $f_{xy}(x,y)$ is not continuous at (0,0).

(2.) What is the relevance of the example above to the theorem about equality of mixed partials?